Relativistic Langevin dynamics

R. J. Fries, R. Rapp, and Collaborators

The approach of heavy-quark (charm, bottom) distributions to thermal equilibrium is one of the outstanding theoretical questions in heavy-ion collisions. While it has been established experimentally that light quarks and gluons thermalize early in the collision, heavy-quark thermalization times are believed to be suppressed by their large masses, implying incomplete thermalization during the lifetime of the fireball. However, measurements of heavy-flavor observables in heavy-ion collisions at RHIC and LHC show a remarkable degree of thermalization, allowing us to study heavy-quark interactions with hot nuclear matter in detail.

On the theoretical side the thermalization process of heavy quarks can be described by the Fokker-Planck (FP) equation which can be obtained from the Boltzmann equation. The drag and diffusion coefficients in the FP equation can therefore be strictly derived from the theory underlying the Boltzmann equation. The commonly used numerical implementation of the Brownian motion is given by the Langevin equation, representing a stochastic realization of the FP equation. Different realizations of the Langevin algorithm exist (post-point, pre-point, etc.) which require special care in how to implement the noise. This is known as the Ito-Stratonovich dilemma which, in the relativistic case has, has posed considerable challenges [1,2]. This rather general problem in transport theory also applies to heavy-quark thermalization in quark gluon plasma.

Starting from the assumptions that the transport coefficients in the Fokker-Planck equation are fixed by the underlying microscopic theory, and that the correct equilibrium distributions are reached in the long-time limit, we have shown that there are unique relations between the drag coefficient Γ in each Langevin implementation and the transport coefficients A and D of the Fokker-Planck equation [3]. If those relations are taken into account the different Langevin implementations give the same result and converge to the correct equilibrium distributions. We have given explicit formulas which set up the correct algorithm for the Langevin equations in pre-point and post-point formulation for non-homogeneous, relativistic background media.

We have furthermore explicitly checked our results numerically with test particles diffusing in a relativistic medium, confirming that the correct equilibrium (Boltzmann-Jüttner) distributions are obtained. Fig. 1 shows the results of Langevin calculations for heavy quarks in a flowing background medium in the pre-point algorithm, using the appropriate relation between the drag coefficient Γ and the diffusion coefficient D (circles and triangles) which lead to the known equilibrium (Boltzmann-Jüttner) distribution (red solid line), while a Langevin calculation violating the relation we found (squares) leads to an incorrect equilibrium distribution (green dotted line). These results have been used in our previously reported calculations of heavy-flavor observables in heavy-ion collisions.

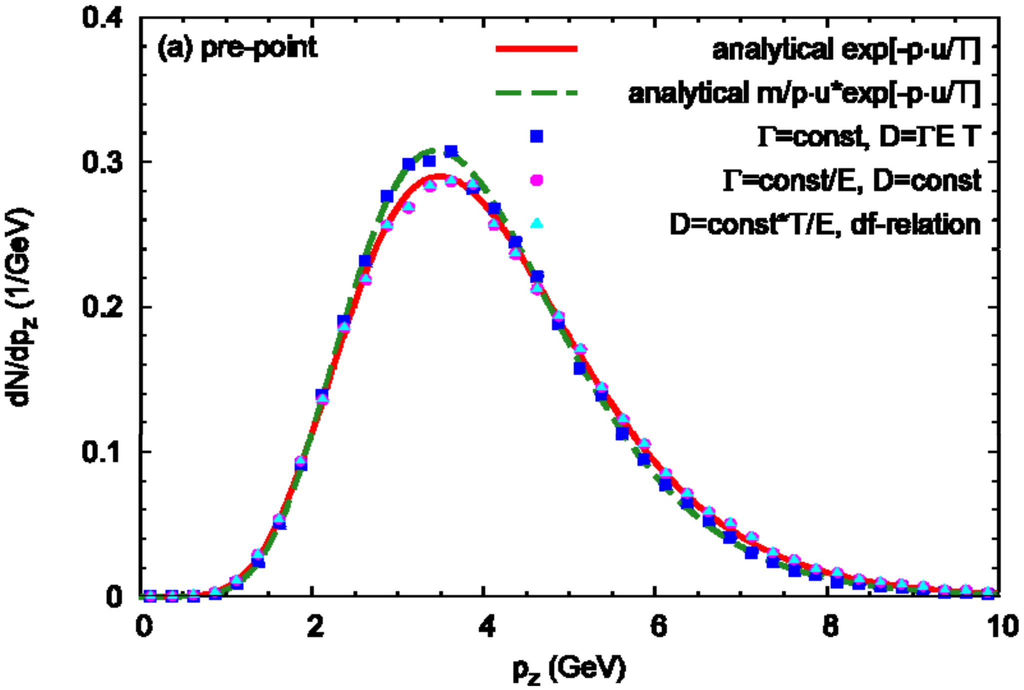

FIG. 1. Langevin simulations in the pre-point scheme for heavy quarks of mass 1.5 GeV in a medium with temperature of T=180 MeV and homogeneous velocity v=0.9 in z-direction. Only simulations with the correct relation between Γ and *D* (circles and triangles) converge to the equilibrium distribution (red solid line).

[1] J. Dunkel and P. Hänggi, Phys. Rep. **471**, 1 (2009).

[2] T. Koide and T. Kodama, Phys. Rev. E **83**, 061111 (2011).

[3] M. He, H. van Hees, P.B. Gossiaux, R.J. Fries and R. Rapp, Phys. Rev. E **88**, 032138 (2013).